A case study of speculative financial bubbles in the South African stock market 2003-2006

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Abstract

We tested 45 indices and common stocks in the South African stock market for the possible existence of a bubble over the period from January 2003 to May 2006. A bubble is defined by a faster-than-exponential acceleration with significant log-periodic oscillations. These two traits are analyzed using different methods. Sensitivity tests shows that the estimated parameters are robust. With the insight of 6 additional month of data since the analysis was performed, we observe that many of the stocks on the South Africa market experienced an abrupt drop mid-June 2006, which is compatible with the predicted t_c for several of the stocks, but not all. This suggests that the mini-crash that occurred around mid-June of 2006 was only a partial correction, which has resumed into a renewed bubbly acceleration bound to end some times in 2007, similarly to what happened on the US market from October 1997 to August 1998.

Key words: Econophysics; financial bubble; super-exponential acceleration; log-periodicity; power-law singularity; African common stocks

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1 Introduction

One of the most robust characteristics of humans, which has arguably the most visible imprint in our social affairs, is imitation and herding. Imitation has been documented in psychology and in neuro-sciences as one of the most evolved cognitive process, requiring a developed cortex and sophisticated processing abilities. In short, we learn our basics and how to adapt mostly by imitation all along our life. It seems that imitation has evolved as an evolutionary advantageous trait, and may even have promoted the development of our anomalously large brain (compared with other mammals) [1]. It is actually "rational" to imitate when lacking sufficient time, energy and information to take a decision based only on private information and processing, that is..., most of the time. Imitation, in obvious or subtle forms, is a pervasive activity of humans. In the modern business, economic and financial worlds, the tendency for humans to imitate leads in its strongest form to herding and to crowd effects [2].

Models of cooperative herding and imitation have been built on the notion that imitation leads to positive feedbacks, that is, an action leads to consequences which themselves reinforce the action and so on, leading to virtuous or vicious circles. We have formalized these ideas in the mathematical theory of rational expectation bubbles in the presence of noisy imitative traders. The main idea is to take into account positive feedbacks, due for instance to derivative hedging, portfolio insurance and foremost to imitative trading, as an essential cause for the appearance of non-sustainable bubble regimes. Specifically, the positive feedbacks give rise to power law (i.e., faster than exponential) acceleration of prices. Previous works by us and our co-workers as well as a few other groups suggest that a robust additional feature characterizes the faster-than-exponential growth of prices during bubbles: the existence of accelerating ups and downs, roughly organized according to a geometrically convergence series of characteristic time scales decorating the power law acceleration. Such patterns have been coined "log-periodic power law" (LPPL).

Several mechanisms are known to generate LPPL, suggesting several complementary non-necessarily exclusive explanations. A dynamical explanation [3, 4] consists in taking into account the competition between positive feedback (self-fulfilling sentiment), negative feedbacks (contrariant behavior and fundamental value analysis) and inertia (everything takes time to adjust). The competition between nonlinear trend followers and nonlinear value investors together with inertia between investor decisions and their market impact may lead to nonlinear oscillations approximating log-periodicity [3, 4]. In this case, log-periodicity is nothing but the observable signature of the developing discrete hierarchy of alternating positive and negative feedbacks culminating in the final "rupture," which is the end of the bubble often associated with a crash. Another explanation is based on the natural hierarchical structure of human groups [5, 6], applied to the network structure of traders [7, 8]. In this class of models, investors in the stock market form a com-

plex connected hierarchical network and interact with each other "locally" through transfers of information, leading to what we refer broadly as "imitation." Local interactions propagate spontaneously into global cooperation leading to herding behaviors, which result in bubbles. These ingredients, together with the fact that prices reflect the aggregate decisions of investors, is formalized by a rational expectation model of bubbles with imitation between the noise traders [7, 8, 9]. The main consequence of these models is that the dynamics may evolve towards a critical point at a critical time t_c corresponding to the most probable end of the bubble. We refer to the book by Sornette [2] for a general introduction, a synthesis of the models and examples of empirical tests and applications.

Mathematically, in its simplest version, the LPPL model is represented by the equation (1) giving the anticipated expected trajectory of the log-price $I(t) \equiv \mathrm{E}[\ln p(t)]$ of a given asset as a function of time, expressed in terms of the distance $\tau = t_c - t$ to the critical time t_c for bubbles (respectively for antibubbles):

$$I(t) = A + B\tau^m + C\tau^m \cos[\omega \ln(\tau) - \phi] \ . \tag{1}$$

A is the expected log-price at t_c (since $\tau = 0$ at $t = t_c$, all the other terms are vanishing at $t = t_c$), B (respectively) controls the amplitude of the power law acceleration (respectively the log-periodic component) of the log-price. The exponent m encodes the structural shape of the acceleration. It is usually found between 0 and 1, which ensures a finite price at t_c together with an asymptotic infinite rate of change close to t_c. The parameter ω is the log-periodic angular frequency of the log-periodic oscillations. It should be stressed that ω is not the inverse of a time scale, but rather it is proportional to the inverse of the logarithm of a scale factor λ, where λ is roughly speaking the ratio of the distances between successive peaks of the log-periodic oscillations. Finally, the phase ϕ contains two ingredients: the information on the mechanism of interactions between investors and a rescaling of time [10, 11].

In a nutshell, the LPPL model (1) describes two phenomena: (i) a faster-than-exponential growth (for $0 < m < 1$ and $B < 0$) of the expected log-price culminating in a singularity of its slope at the critical time t_c and (ii) an accelerating sequence of local rallies and corrections decorating the overall power law acceleration (the "log-periodicity"). The literature cited above contains several derivations of this equation (1) and we refer the reader to them. Basically, the power law part $A + B\tau^m$ embodies the effect of collective imitation leading to global herding and a critical behavior characterized mathematically by the singularity at t_c (in a way reminiscent to phase transitions in physics, but here in the time domain). The log-periodic component reflects at least two possible effects as mentioned above: (1) an inherent discrete hierarchical structure in the social network of investors and (2) a nonlinear mean reversal behavior of fundamental investing styles. See Ref. [12] for a general review on the symmetry of "discrete scale invariance" at the basis of log-periodicity.

Table 1
The ticker codes and corresponding numbering of 45 South African stocks (http://www.jse.co.za/) analyzed in the present work.

Num	1	2	3	4	5	6	7	8	9
Code	J203	J210	J257	J580	ABL	AGL	AMS	ANG	APN
Num	10	11	12	13	14	15	16	17	18
Code	ASA	BAW	BIL	BVT	ECO	FSR	GFI	HAR	IMP
Num	19	20	21	22	23	24	25	26	27
Code	INL	INP	IPL	JDG	KMB	LBT	LGL	MLA	MTN
Num	28	29	30	31	32	33	34	35	36
Code	NED	NPN	NTC	OML	PIK	PPC	RCH	REM	RMH
Num	37	38	39	40	41	42	43	44	45
Code	SAB	SAP	SBK	SHF	SLM	SOL	TBS	TKG	WHL

The organization of the paper is as follows. The next section 2 tests for the existence of faster-than-exponential acceleration in the price of the 45 stocks used in this study (their ticker code used in the South African stock exchange and the corresponding numbering we use are given in Table 1). It also presents preliminary screening tests on the existence of log-periodicity. Combining these evidences, section 2 finally qualifies 5 assets out of the 45 as exhibiting a significant bubble regime. Then, section 3 presents a detailed analysis of the log-periodic characteristics of these 5 assets, using four different techniques. The estimations of the angular log-frequency ω's are found consistent and robust across the four methods. Section 4 presents a sensitivity analysis of the critical time t_c and of the angular log-frequency ω by varying both the starting time as well as the ending time of the interval over which the fits are performed. Section 5 summarizes and concludes with respect to the quality of the forecasts using t_c as the most probable time of the corrections associated with the end of the bubbles, using the insight obtained from waiting an additional 6 months since the end of the study in May 2006.

2 Classification of potential speculative behavior among 45 representative stocks on the Johannesburg Stock Exchange

One of the most advanced and productive economies in Africa, South Africa is characterized by a developed first world economic infrastructure and an emerging market economy. Its financial market is organized by the Johannesburg Stock Exchange (JSE) (http://www.jse.co.za/), which is the only securities exchange in

South Africa. It was officially established on 8 November, 1887, and has now more than 400 listed companies [13].

Our study is performed on a subset of 45 companies among the largest companies listed on the JSE, whose ticker codes are given in table 1 (for detailed information, see the extended version only available online at http://arXiv.org/abs/physics/0701171). The selection of these 45 companies is representative of the diversification performed by major investment banks and brokerage houses in South Africa, as communicated to us by one of the major brokerage house in South Africa. The daily price series run from the first trading day of January 2003 till May 2006.

2.1 Is there an acceleration?

Our first goal is to perform a robust analysis of the the 45 financial time series, to identify those potential candidates for a bubble behavior. There is a large literature on the empirical issue of identifying financial bubbles (see [2, 14, 15, 16, 17, 18, 19, 20, 21, 22] and references therein). A key problem is that bubbles are usually defined as exponentially explosive growth phases, but how can one then distinguish them from the growth of a fundamental valuation process which is also generically expected to follow an exponential growth path? We address this problem by defining a bubble as a faster-than-exponential accelerating price [2, 22], which we refer to as "super-exponential." Being faster than exponential, i.e., the growth rate is itself growing, it is necessarily unsustainable if we assume a standard geometric growth for the underlying economy. A super-exponential path can only be transient, reflecting various positive feedback processes that lead to reinforced growth.

Our goal is thus to contrast a standard exponential growth from a super-exponential growth. The former is characterized by the logarithm of the price of a given asset being linear in time as

$$\ln[p(t)] = a + bt + \epsilon_1 \ , \tag{2}$$

where ϵ_1 is a short-hand notation for a random walk component. The parameter b is then the rate of return for continuous compounding of the interests. Expression (2) is nothing but the integrated form of the standard geometrical random walk model with drift. The arguably simplest extension (2) which gives super-exponential growth is obtained as

$$\ln[p(t)] = a + bt + ct^2 + \epsilon_2 \ . \tag{3}$$

The null hypothesis is $c = 0$. If it is rejected with $c > 0$ at a sufficiently large significance level, we would conclude that the price process is growing super-exponentially, qualifying a bubble regime. Since the residual is not Gaussian and not stationary, we cannot use directly the Wilks' likelihood ratio test or t-test of significance for the coefficient c. We rather use an approach in the spirit of the Akaike

Information Criterion (AIC), which amounts to test the two models by comparing $[RMS_1(i)]^2$ for model (2) with $[RMS_2(i)]^2 + 2\sigma^2$ for model (3) [23]. The terms $RMS_1(i)$ and $RMS_2(i)$ are the root-mean-square of the residuals of the fits to the price time series of asset i with model (2) and model (3) respectively. More precisely, the calibration of model (2) determines the best values of the parameters a and b for each stock i and $RMS_1(i)$ then provides a measure of the average standard deviation of the residual ϵ_1. Similarly, the calibration of model (3) determines the best values of the parameters a, b and c for each stock i and $RMS_2(i)$ then provides a measure of the average standard deviation of the residual ϵ_2. The term σ^2 is the square of the standard deviation of the noise of the price process. The term $2\sigma^2$ is the cost attributed to model (3) for having one more adjustable parameter compared with model (2). In other words, AIC not only rewards goodness of fit, but also includes a penalty linear in the number of estimated parameters. Then, the AIC qualifies model (3) if $[RMS_2(i)]^2 + 2\sigma^2 < [RMS_1(i)]^2$. This is equivalent to

$$\frac{[RMS_1(i)]^2 - [RMS_2(i)]^2}{[RMS_1(i)]^2} > \frac{2\sigma^2}{[RMS_1(i)]^2} \ . \tag{4}$$

The ratio in the l.h.s. of inequality (4) quantifies the relative goodness of fit of expressions (2) and (3). Since in practice, σ^2 is not known to us, we calculate the relative difference of the improvement of the fits resulting from the additional quadratic term in model (3) in the left-hand-side of (4):

$$D(i) = [RMS_1(i) - RMS_2(i)]/RMS_1(i) \ , \tag{5}$$

for each asset. Then, the larger is the value of $D(i)$, the more probable is the rejection of the null hypothesis and the relevance of the quadratic term qualifying a bubble regime. Here, we put aside the subtle issues [24, 25] stemming from possible spurious regressions resulting from the non-stationarity of the residuals in models (2) and (3), which are left for another work. Fig. 1 gives the 45 values of $D(i)$ for all the 45 stocks.

Fig. 1 suggests the existence of two clusters, characterized by values of the "Relative RMS change" above or below $\approx 25\%$. We thus disqualify stocks as not being in a bubble regime if they obey at least one of the following criteria:

(1) $D(i) \leqslant 25\%$,
(2) the quadratic term is positive ($c > 0$),
(3) the overall price has been increasing from the beginning to the end of the period.

This leads us to reject the following 27 stocks as not in a bubble regime from Jan. 2003 till May 2006: 3, 5, 9, 10, 11, 12, 13, 14, 15, 21, 22, 24, 26, 27, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45. This selection is in line with visual inspection for all except stock 30, for which the analysis comparing (2) and (3) has not enough power. Indeed, in this case of stock 30, the weak improvement of model (3) may

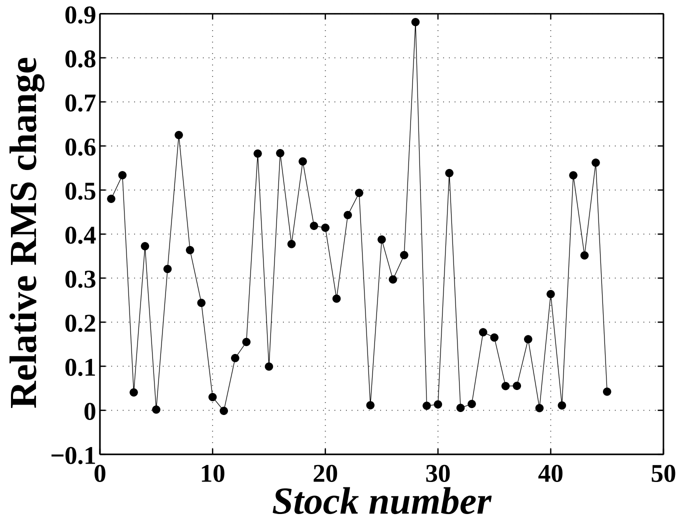

Fig. 1. Relative difference of the improvement of the fits resulting from the additional quadratic term in model (3) in the left-hand-side of (4), as defined by (5).

be attributed to the combination of a large drawup followed by a large drawdown from end-of-2003 to middle-of-2004 followed by an upward acceleration. We thus keep this stock 30 for further analysis.

2.2 Log-periodic oscillations

As we just mentioned, the analysis comparing (2) and (3) is not claimed to have universal absolute power: it may have not enough power to reject a stock when it should have been rejected (error of type I or false positive), or it may reject a stock that should be kept in the bubble class (error of type II or false negative) as we argue is perhaps the case of stock 30.

It is thus useful to examine the results of another test, based on a specification which is well-adapted to test for the presence of super-exponential behavior, in the possible presence of oscillatory intermittent fluctuations. This specification uses the so-called log-periodic formula (1) introduced first in [26, 27] and expanded upon in several subsequent papers (see for instance [2, 28]. We have fitted the 45 stock price series with the log-periodic power-law formula (1) and obtained the key parameters t_c, m, and ω.

In the above characterization in terms of (2) versus (3), the non-accelerating log-price was characterized essentially by the absence of a significant positive quadratic term ct^2 in (3). In the present log-periodic power law analysis, a non-accelerating log-price should be qualified by an exponent m close to 1, while an accelerating log-price corresponds to m small or even negative. We find that this correspondence holds to a large degree, as seen from the following list: (3, 0.86), (5, 0.96), (9, 0.52),

(10, 0.86), (11, 0.84), (12, 0.58), (13, 0.84), (14, 2), (15, 0.71), (21, 0.27), (22, 1.6), (24, 0.5), (26, 1.6), (27, 1.8), (29, 0.99), (32, 1), (33, 0.63), (34, 0.41), (35, 0.84), (36, 1.4), (37, 1.3), (38,-2), (39, 0.99), (41, 0.64), (44, 1.4), (45, 0.55). Here, the first number in each parenthesis stands for the stock number and the second one is the value of the exponent m. For the stock prices which have been qualified as non-accelerating by the method of the previous section, we find values of m typically larger than 0.8, confirming the classification. The following cases can be considered marginal:

(1) (9, 0.52), which has a relative RMS change $D(9) \approx 0.24$ as defined by (5),
(2) (12, 0.58), which has a relative RMS change $D(12) \approx 0.12$,
(3) (21, 0.27), which has a relative RMS change $D(21) \approx 0.24$,
(4) (24, 0.5), which has a relative RMS change $D(24)$ of almost zero, while presenting an intermediate m value,
(5) (34, 0.41), which has a relative RMS change $D(34) = 0.18$ and the acceleration is more an abrupt change of slope or kink occuring in the first quarter of 2005, and
(6) (38, -2), which exhibits a log-price which has decreased over most of the time period and which should be rejected.

In addition to the value of the exponent m, the presence of log-periodicity has been argued as a signature of a bubble regime [26, 27]. The upper panel of Fig. 2 plots the value of the fitted log-periodic angular frequency ω for the 45 analyzed stocks. Previous studies on a large number of bubbles mostly on financial indices, bonds and currencies of many developed and emerging countries have shown that the distribution of ω exhibits several peaks, the prominent one being on the so-called fundamental log-periodic angular frequency $\omega_1 \approx 6.4 \pm 1.5$ [29], with other peaks on its harmonics $\omega_n = n\omega_1$. The importance of the high-order harmonics is expected to decrease exponentially [30], but large amplitudes for the second-order and third-order harmonics ω_2 and ω_3 have been observed to be sometimes very significant [31, 32]. It thus seems difficult to use here a filter based solely on ω, in particular for individual stocks which are necessarily more noisy than aggregate indices.

Ref. [33] has investigated all kinds of scaling series (Bm, fBm, Levy model) to establish the statistical significance level of periodicity and log-periodicity in noisy time series. Ref. [7] also reports extensive tests with GARCH processes to assess the statistical significance level of log-periodicity. Our systematic statistical analysis of the significance level of periodic (as well as log-periodic) signals performed in Ref. [33] shows that the hypothesis, that the observed log-periodicity results from noise, can be rejected at a confidence level higher than 95% as soon as the the number $N_{\rm osc}$ of oscillations is 3 or more, for most types of noises. For instance, it has been shown that multiplicative noise on a power law accelerating function leads naturally to stochastic log-periodic oscillations with a most probable number equal to $N_{\rm osc} \approx 1.5$ [34]. The tests of statistical significance performed in [33] have

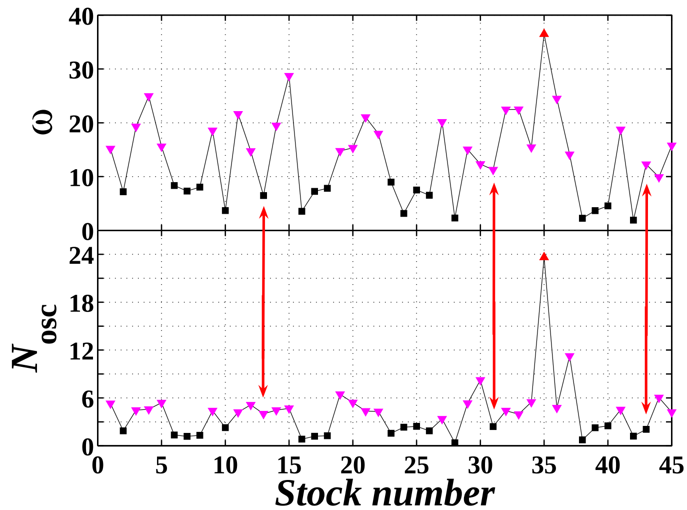

Fig. 2. (Color online). Classification of the 45 analyzed South African stocks with respect to the properties of their log-periodic oscillations. Upper panel: The angular logarithmic frequency ω for each of the 45 stock indexed with its number i. Stocks having $\omega \leqslant 9$ are marked with solid squares, stocks with $9 < \omega \leqslant 30$ are indicated by solid downward triangles, and stocks with $\omega \geqslant 30$ are shown with a solid upward triangle. Lower panel: The number of oscillations $N_{\rm osc}$ over which log-periodicity is found, as a function of the stock number i. Stocks having $N_{\rm osc} \leqslant 3$ are marked with solid squares, those with $3 < N_{\rm osc} \leqslant 10$ are indicated by solid downward triangles, and the stocks with $N_{\rm osc} \geqslant 10$ are shown with a solid upward triangle.

shown that, for most types of noise, three oscillations are in general sufficient to qualify a genuine oscillatory component. Rigorously, by "genuine", we mean that the probability that the observed oscillatory behavior results from some random noise configuration rather than from some informative signal is below a standard significance level, typically $1 - p$ with $p = 95\%$ or 99%. We thus complement the determination of the angular log-frequency ω by the measure $N_{\rm osc}$ of log-periodic oscillations in each of the 45 stock prices. Given a LPPL fit and the obtained calibrated parameters t_c and ω, the number of oscillations is determined by

$$N_{\rm osc} = \frac{\omega}{2\pi} \ln \left| \frac{t_c - t_{\rm first}}{t_c - t_{\rm last}} \right| , \tag{6}$$

where $[t_{\rm first}, t_{\rm last}]$ is the interval over which the LPPL fitting is performed. The variable $N_{\rm osc}$ for each of the 45 stocks is shown in the lower panel of Fig. 2.

One can observe that the two measures ω and $N_{\rm osc}$ provide consistent results. As shown by the correspondence between the symbols in the upper and lower panel of Fig. 2 for most of the stocks, large ω's are associated with a large number of oscillations and vice-versa. There are three exceptions to this observation: $i = 13$ (BVT), 31 (OML), and 43 (TBS), outlined in the figure by the arrows. Given the large range of observed ω's, we take the number $N_{\rm osc}$ of LPPL oscillations as the more robust indicator of genuine log-periodicity. This leads to qualify the following

stocks as exhibiting a significant log-periodicity: 1, 3, 4, 5, 9, 11, 12, 13, 14, 15, 19, 20, 21, 22, 27, 29, 30, 32, 33, 34, 36, 37, 41, 44, and 45.

Combining this with the results shown in Sec. 2.1, we are left with five stocks which qualify as being in a bubble regime, defined as a significant super-exponential acceleration with the presence of significant log-periodicity: 1 (J203: an index), 4 (J580: an index), 19 (INL), 20 (INP), and 30 (NTC).

In the remaining of this paper, we analyze these five stocks extensively.

3 Analyzing the log-periodic structure of stocks 1 (J203), 4 (J580), 19 (INL), 20 (INP), and 30 (NTC)

3.1 First-order LPPL model

The fits of the logarithms of the prices of the five stocks 1 (J203), 4 (J580), 19 (INL), 20 (INP), and 30 (NTC) by the log-periodic power law model (1) are shown in Fig. 3 and the parameters are given in Table 2. The small value of the exponent m (except for NTC) confirms a clear super-exponential acceleration. The log-periodic angular frequencies are found to be close to twice (J203, INL, INP, NTC) or four times (J580) the value $\omega_1 \approx 6.4 \pm 1.5$ of the fundamental log-periodic angular frequency found in many previous studies [22, 29]. The LPPL fits suggested at the time of the fits (end of May 2006) that the bubbles would end either immediately (NTC) or during the second part of the year. We discuss this prediction below.

Table 2
Parameters of the first-order log-periodic power law model providing the best fits to the the five South African stocks 1 (J203), 4 (J580), 19 (INL), 20 (INP), and 30 (NTC) shown in Fig. 3

Stock	t_c	m	ω	ϕ	A	B	10^3C	χ
J203 (1)	2006/11/01	0.27	15.3	5.41	11.3	-0.336	8.73	0.036
J580 (4)	2007/11/12	0.30	24.1	4.20	12.4	-0.388	4.84	0.040
INL (19)	2006/08/07	0.37	14.1	1.89	11.3	-0.159	8.08	0.072
INP (20)	2006/09/19	0.30	14.0	4.81	11.9	-0.307	-11.27	0.074
NTC (30)	2006/05/17	0.60	12.8	2.21	6.9	-0.014	-1.91	0.054

The detection of log-periodic oscillations, if any, is conveniently performed by removing the global trend of the price of a given stock. One way is to subtract the power law trend from the price and then to analyze the wobbles of the obtained residuals $s(t)$ by an adequate spectral analysis [9]. We shall also use a non-

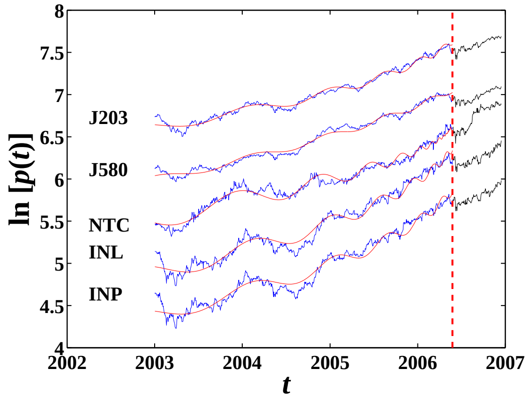

Fig. 3. (Color online). Fits of the first-order LPPL model (1) to the logarithm of the five South African stock prices 1 (J203), 4 (J580), 19 (INL), 20 (INP), and 30 (NTC) from January 2003 to May 2006. The vertical dashed line indicated the end of the fitting interval: May 2006. The rough curves are the historical raw data and extends till Dec. 2006 and the smooth curves are the LPPL fits. The data from June to Dec. 2006 to the right of the vertical dashed line has not been used in the fits. The values of the fit parameters are listed in Table 2. The curves have been shifted vertically for clarity.

parametric approach called the (H, q)-analysis [35]. Since log-periodicity corresponds to regular oscillations in the variable $\ln(t-t_c)$, we use a Lomb periodogram analysis which is well-adapted to the uneven sampling of the variable $\ln(t-t_c)$ [36]. The Lomb periodogram analysis is nothing but a spectral analysis based on a least squares fit of sinusoids to data samples, similar to Fourier analysis. The Lomb analysis also allows us to assess the statistical significance level of the extracted log-periodicity [33, 36, 37].

3.2 Parametric detrending approach

Following [7, 9], the first method of analysis of the log-periodicity consists in removing the power law trend and then testing for a possible pure log-periodicity without acceleration. We construct the residual $s(t)$ in the following way

$$s(t) = [I(t) - A]/(t - t_c)^m \ . \tag{7}$$

This residual $s(t)$ has a nonzero mean μ_s associated with the coefficient B and a given variance σ_s^2 (in general different from 1). The inset of Fig. 4 plots the standardized residuals $[s(t)-\mu_s]/\sigma_s$ as a function of $\ln \tau = \ln(t-t_c)$ for the five stocks 1 (J203), 4 (J580), 19 (INL), 20 (INP), and 30 (NTC). The five Lomb periodograms of these five residuals are presented in Fig. 4. Since $P_N(\omega)$ is a normalized Lomb power, $s(t)$ and $[s(t) - \mu_s]/\sigma_s$ have identical Lomb periodogram. The angular log-periodic frequencies associated with the highest Lomb peaks are respectively 15.1, 25.1, 15.0, 15.6, and 12.5, which are close to the values obtained with the parametric fit with expression (1) listed in Table 2. Their statistical significance is ex-

tremely high, much larger than 99%, for all possible noise processes, according to the benchmarks developed in [33].

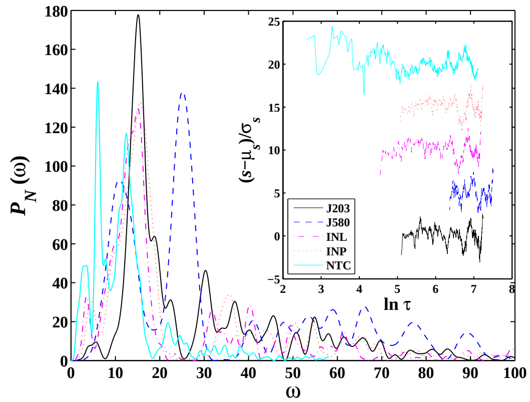

Fig. 4. (Color online). Lomb periodogram analysis of the five standardized residuals $[s(t) - \mu_s]/\sigma_s$ shown in the inset, where $s(t)$ is defined by (7), considered as a function of $\ln(\tau) = \ln(t - t_c)$. The curves in the inset have been translated vertically for clarity.

3.3 (H, q)-analysis

We have also performed a generalized q-analysis, called (H, q)-analysis [35, 38], on each of the logarithm of the five price trajectories. The (H, q)-analysis is a non-parametric method for characterizing self-similar functions, which generalizes the q-analysis [39, 40]. The later is a natural tool for the description of discretely scale invariant fractals. The (H, q)-derivative of a function $I(\tau)$ is defined as

$$D_q^H I(\tau) \triangleq \frac{I(\tau) - I(q\tau)}{[(1-q)\tau]^H} . \tag{8}$$

The special case $H = 1$ recovers the standard q-derivative, which itself reduces to the standard derivative in the limit $q \to 1^-$. There is no loss of generality by constraining q in the open interval $(0, 1)$ [35]. The advantage of the (H, q)-analysis is that there is no need for detrending, as trends are automatically accounted for by the finite difference and the normalization by the denominator upon a systematic sweeping of the parameter H.

We apply the (H, q)-analysis to $I(x) = \ln p(t)$, where $p(t)$ is the price of each of the five stocks 1 (J203), 4 (J580), 19 (INL), 20 (INP), and 30 (NTC), as an independent powerful test of log-periodicity. The independent variable is taken to be $\ln \tau$ [35]. The same method has been applied to test for log-periodicity in stock market bubbles and antibubbles [38, 41], in the USA foreign capital inflow bubble ending in early 2001 [42], in the UK real estate bubble [43], in the Chinese stock market antibubble [44], and in the US treasury bond yield antibubble [45].

We scan a 100×50 rectangular grid in the (H, q) plane, with $H = -0.99 : 0.02 :$

0.99 (from -0.99 to $+0.99$ with increment 0.02) and $q = 0.01 : 0.02 : 0.99$ (from 0.01 to 0.99 with increment 0.02). For each pair of (H, q) values, we calculate the (H, q)-derivative (8), on which we perform a Lomb analysis. The highest Lomb peak of the resulting periodogram has height P_N and abscissa ω, both P_N and ω being functions of H and q. Figure 5 shows the numerically constructed discrete binned bivariate distribution of pairs (ω, P_N) for $12 \leqslant \omega \leqslant 17$. For values $\omega \leqslant 12$, only one cluster with small values $\omega < 4.75$ can be observed: such small values of ω are associated with only one or at most two oscillations and correspond most probably to the most probable oscillatory structure of multiplicative noise of power law function [34]. Another possible origin of this cluster at $\omega < 4.75$ is a residual global trend which has not been completely accounted for by the (H, q)-derivative [35, 38].

Most of the Lomb periodograms associated with the points drawn in Fig. 5 have a shape similar to that shown in Fig. 4. Specifically, the average angular log-frequencies determined from this (H, q) analysis are 14.7 ± 0.3 for stock "J203", 14.7 ± 0.1 for stock "J580", 15.0 ± 1.3 for stock "INL", 13.7 ± 1.5 for stock "INP", and 15.4 ± 0.1 for stock "NTC". In summary, this (H, q)-analysis provides even stronger evidence for the existence of log-periodicity than the parametric detrending approach of the previous section 3.2.

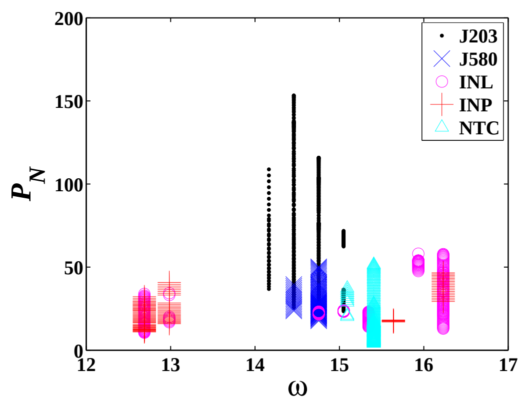

Fig. 5. (Color online). Numerically constructed discrete binned bivariate distribution of pairs (ω, P_N), defined as the angular log-frequencies and corresponding highest Lomb peaks of the (H, q)-derivative of $\ln[p(t)]$, when scanning a 100×50 rectangular grid in the (H, q) plane, (H from -0.99 to $+0.99$ with increment 0.02 and q from 0.01 to 0.99 with increment 0.02). Each marker in the figure corresponds to the highest Lomb peak and its associated angular log-frequency in the Lomb periodogram of the (H, q)-derivative of $\ln[p(t)]$ for a given pair (H, q).

3.4 Second-order Weierstrass-type LPPL model

As already pointed out above, the log-periodic angular frequencies are found to be close to twice (J203, INL, INP, NTC) or four times (J580) the value $\omega_1 \approx 6.4 \pm 1.5$

of the fundamental log-periodic angular frequency found in many previous studies [22, 29]. This suggests that the first-order LPPL formula (1) used until now should be extended to include several harmonics. Indeed, the general mathematical formulation of discrete scale invariance [12, 28] shows that a log-periodic function is expected in general to be represented by a systematic series of log-periodic terms of the form [30]

$$I(t) = A + B\tau^m + \Re\left(\sum_{n=1}^{N} C_n e^{i\psi_n} \tau^{-s_n}\right) , \tag{9}$$

where $\tau = t_c - t$ as before. The main advantage of the high-order Weierstrass-type LPPL models is that they allow to identify the fundamental log-frequency and its harmonics. This family of models have been applied to the case study of many bubbles and antibubbles [32], such as the UK real estate bubble in the last decade [43], the 1975-2001 bubble in the American foreign assets capital inflow [42], the Chinese stock market antibubble since 2001 [44], and the USA treasury bond yield antibubble since 2000 [45].

We adopt the second-order Weierstrass-type LPPL model (9) with $N = 2$ to fit the logarithm of the five South African stock prices. The time evolution of the five stocks and the corresponding fits are drawn in Fig. 6.

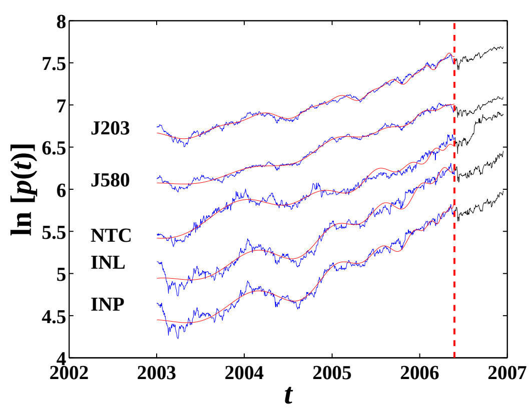

Fig. 6. (Color online). Best fits of the second-order Weierstrass-type LPPL formula (9) with $N = 2$ to the logarithm of the five South African stock prices from January 2003 to May 2006. The rough curves are the historical raw data and the smooth curves are the associated LPPL fits. The values of the fit parameters are listed in Table 3. The curves have been shifted vertically for clarity.

The parameters of the five fits using the second-order Weierstrass-type LPPL model are presented in Table 3. We observe that, except from stock J203, the angular log-frequencies lie in the range $6.5 \leq \omega \leq 8.1$, which is consistent with previous results on the fundamental log-frequency $\omega_1 = 6.4 \pm 1.5$ [22, 29]. Specifically, this confirms the coexistence of this fundamental log-frequency together with its harmonics, justifying the interpretation of the large values reported above with the first-order formula (1) as corresponding to the harmonics of ω_1. The fact that the

angular log-frequency $\omega = 15.4$ for stock "J203" is close to the second-order harmonic of ω_1 is probably associated with a very strong amplitude of the second harmonics, which may hide the existence of ω_1. It is also interesting to notice that the absolute values of the linear parameters C_1 and C_2 are comparable. In three cases $|C_2| > |C_1|$, indicating that the amplitudes of the second-order harmonic oscillations are considerable, again consistent with our previous interpretation of the results obtained above.

Except for NTC, the exponents m are larger in the second-order LPPL fit than in the first-order case. The critical times t_c predicted to be the end of the bubbles are quite robust: they are essentially unchanged for J203 and J580 while they are pushed towards the future by roughly three months for the three other stocks, when going from the first-order to the second-order formula.

Table 3
Parameters of the fits with the second-order Weierstrass-type LPPL model (9) of the five South African stocks

Stock	t_c	m	ω	ϕ_1	ϕ_2	A	B	10^3C_1	10^3C_2	χ
J203	2006/11/05	0.27	15.4	5.71	3.2	11.3	-0.336	8.51	4.16	0.032
J580	2006/10/27	0.68	7.5	4.14	3.3	10.2	-0.010	0.53	0.47	0.029
INL	2006/11/11	0.39	8.1	4.24	2.4	11.7	-0.156	-4.11	-6.72	0.066
INP	2006/06/22	0.61	6.5	1.58	6.2	10.7	-0.021	1.39	1.87	0.062
NTC	2006/08/26	0.51	7.2	5.20	5.3	7.2	-0.036	1.98	2.44	0.050

3.5 Comparison of the different methods

Let us now compare the estimated angular log-frequencies of the five stock bubbles obtained with the different methods presented above: (i) the fit with the first order LPPL function, (ii) the parametric detrending approach, (iii) the (H, q)-analysis, and (iv) the fit with the second-order Weierstrass-type function. The obtained angular log-frequencies are listed in Table 4. The results are self-consistent in the sense that all the bubbles have the same fundamental angular log-frequency $\omega = 7.6 \pm 1.9$ and the large values are its higher-order harmonics.

For J203, the four methods give essentially the same value $\omega \approx 15$, which can be interpreted as a very strong second harmonic $2\omega_1$ of the fundamental log-periodic angular log-frequency $\omega_1 \approx 6.4 \pm 1.5$ found in many previous studies [22, 29].

For J580, the second-order LPPL fit correctly identifies the presence of ω_1, and of course its second-order harmonics. The (H, q) analysis identifies the second har-

Table 4
Comparison of the angular log-frequencies of the five South African stock bubbles estimated by the four different methods.

Stock	LPPL	Detrending	(H,q)-analysis	Weierstrass 2
J203	15.3	15.1	14.7 ± 0.3	15.4
J580	24.1	25.1	14.7 ± 0.1	7.5
INL	14.1	15.0	15.0 ± 1.3	8.1
INP	14.0	15.6	13.7 ± 1.5	6.5
NTC	12.8	12.5	15.4 ± 0.1	7.2

monics $2\omega_1$ while the two other methods seem to be most sensitive to the fourth-order harmonics.

For the three other stocks, the second-order LPPL fit correctly identifies the presence of ω_1, while the three other methods extract its second harmonics $2\omega_1$ as being the dominant contribution, in agreement with the amplitude $C_2 > C_1$ determined in their second-order LPPL fit.

We conclude that genuine log-periodicity exists the price trajectories of these five stocks with high statistical significance, and that the extracted values of the angular log-frequencies are compatible with previous results on other bubbles [22, 29]. The main novelty lies in the importance of the second-order and fourth-order harmonics, which is larger than usual.

4 Sensitivity analysis of the critical times t_c

The determination of the critical time t_c is particularly important since it gives the estimated termination time of the bubbles, which can occur approximately two times out of three in the form of a significant correction or a crash. It is noteworthy to stress that a bubble does not end necessarily with a crash as there is a finite probability for a bubble to end with a transition to another regime such as slow deflation or correction [7, 9]. The critical time t_c is thus the end of the LPPL bubble and the time at which the crash is most probable, if it ever occurs.

Given its high significance, we have performed a sensitivity analysis of t_c for the five stocks with respect to different starting time $t_{\rm first}$ and ending time $t_{\rm last}$ of the price time series used in the fitting procedure, following previous works [43, 46, 47].

4.1 *The impact of* t_{first}

We first study the impact of t_{first} on t_c and ω to check the stability of the estimated critical time and the significance of the log-periodic pattern. For each stock, we use the first-order LPPL formula (1) (respectively the second-order LPPL formula (9)) to fit the price time series from t_{first} to $2006/05/26$, where t_{first} ranges from $2003/01/02$ to $2004/12/01$ with a spacing of 20 trading days. The results are presented in Fig. 7 (respectively Fig. 8). The overall conclusion is that both t_c and ω are very robust with respect to the choice of the starting time t_{first} of the fitting interval.

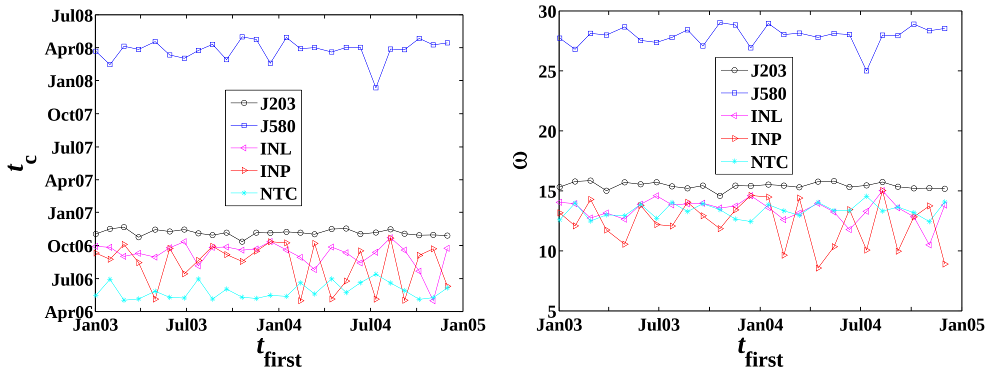

Fig. 7. (Color online). Sensitivity analysis of the estimated critical time t_c and the angular log-frequency ω for the five stocks obtained by varying the last point t_{first} of the time series up to which the fits using formula (1) are performed.

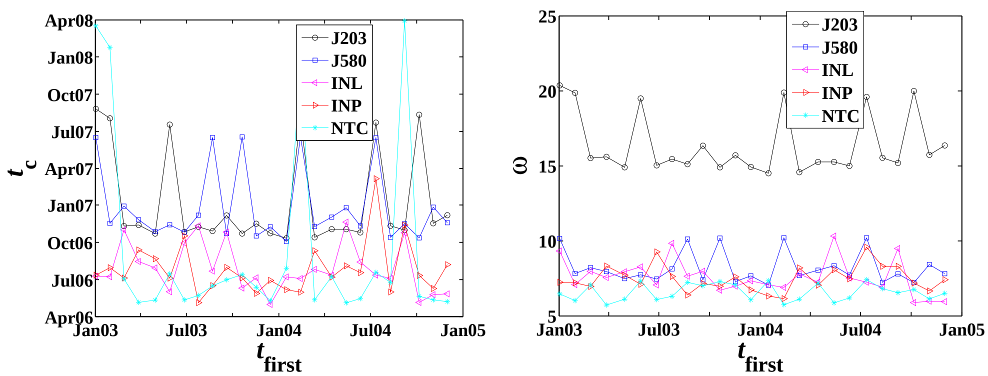

Fig. 8. (Color online). Sensitivity analysis of the estimated critical time t_c and the angular log-frequency ω for the five stocks obtained by varying the last point t_{first} of the time series up to which the fits using formula (9) with $N = 2$ are performed.

4.2 *The impact of* t_{last}

We then study the impact of t_{last} on t_c and ω to check the stability of the estimated critical time and the significance of the log-periodic pattern. For each stock, we

use the first-order LPPL formula (1) (respectively the second-order LPPL formula (9)) to fit the price time series from $2003/01/02$ to $t_{\rm last}$, where $t_{\rm last}$ ranges from $2003/01/02$ to $2004/12/01$ with a spacing of 20 trading days. The results are presented in Fig. 9 (respectively Fig. 10). Overall, the conclusion is similar than for the dependence on $t_{\rm first}$, confirming the robustness of the fits and the reliability of our conclusions. The stock NTC is the only one exhibiting a change of regime in the first quarter of 2006, at which the fitted t_c jumps from Jan. 2006 to May 2006.

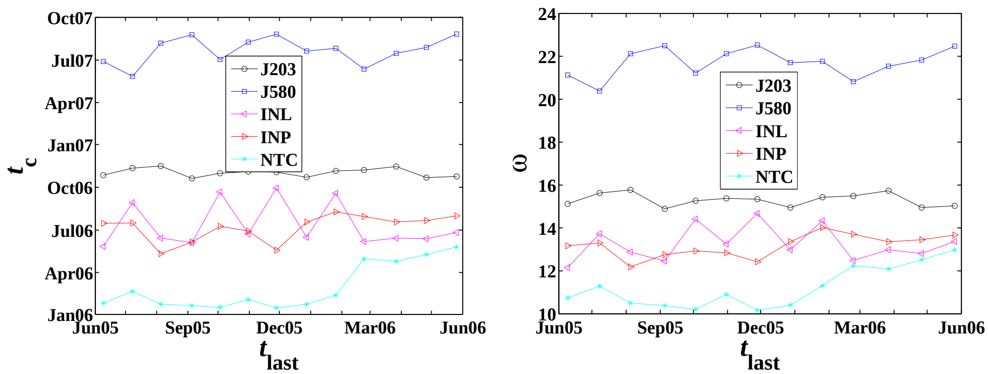

Fig. 9. (Color online). Sensitivity analysis of the estimated critical time t_c and the angular log-frequency ω for the five stocks obtained by varying the last point $t_{\rm last}$ of the time interval in which the fits using formula (1) are performed.

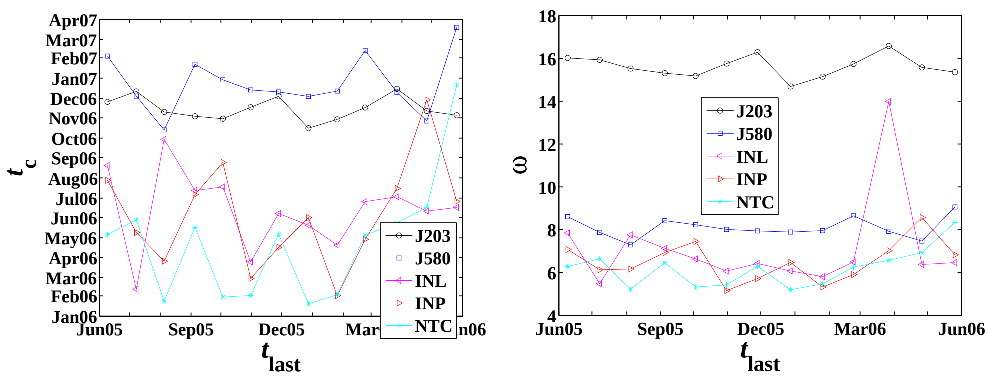

Fig. 10. (Color online). Sensitivity analysis of the estimated critical time t_c and the angular log-frequency ω for the five stocks obtained by varying the last point $t_{\rm last}$ of the time series up to which the fits using formula (9) with $N = 2$ are performed.

5 Concluding remarks

In summary, we have identified five stocks (1 (J203), 4 (J580), 19 (INL), 20 (INP), and 30 (NTC)) out of a representative sample of forty five South African stocks, that we qualified as being in a bubble regime defined as a super-exponential growth regime from Jan. 2003 to May 2006 with significant log-periodic oscillations. We

studied the log-periodic characteristics of these stocks using four different techniques, the parametric fits with the first-order LPPL formula, with the second-order Weierstrass-type model, the parametric detrending method, and the (H, q)-analysis. The four techniques give consistent estimations for the value of the fundamental angular log-frequency ω_1 in agreement with previous works on many other bubbles in developed and emergent markets, confirming with very high statistical confidence the existence of genuine log-periodicity.

Sensitivity tests of the estimated critical times and of the angular log-frequency by varying the first date and the last date of the stock price time series over which the fits are performed confirm the robustness of the estimated parameters.

This study was performed at the end of May 2006 and we waited another six months before completing this paper to see what were the subsequent evolutions of the five stocks. It turns out that the five selected stocks on the South Africa market experienced an abrupt drop in mid-June 2006, as can be seen from Fig. 3 in which we have shown the subsequent price evolution after May 2006. Quantitatively, the cumulative drawdown (defined as the peak-to-valley relative price variation) measured from the highest price value prior to the end of May 2006 to the price bottom thereafter are respectively: 16.8% for 1 (J203), 16.6% for 4 (J580), 20.1% for 19 (INL), 17.9% for 20 (INP), and 20.1% for 30 (NTC). These drawdowns occurred over a time period of less than one month and their amplitudes belong to the 1-percentile of the distribution of drawdowns [48, 49, 50]. The occurrence of these large market price corrections and their timing (mid-June 2006) are compatible with the predicted t_c for INL, INP and NTC and to a lesser extend for J203. However, other stocks including J580 give a much large t_c, some time during 2007, suggesting that the potential for growth in several of these stocks is not exhausted. It is possible that the mini-crash that occurred in mid-June 2006 was only a partial correction, similarly to the Oct. 1997 8% drop witnessed on the S&P500 US market which, after being followed by a plateau of three months, resumed in a strong acceleration, to finally end with the real crash in August-September 1998 (see discussion of this sequence in [2]). Finally, we refer to the extended version only available online at http://arXiv.org/abs/physics/0701171, in which the 45 price trajectories for the 45 assets and the fit of the logarithm of their price with the linear model and with the nonlinear model are shown.

Acknowledgments:

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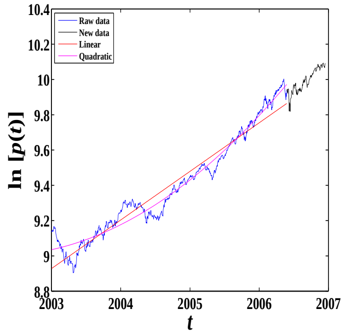
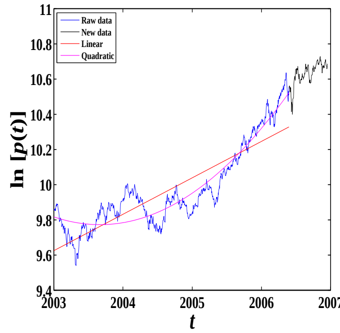
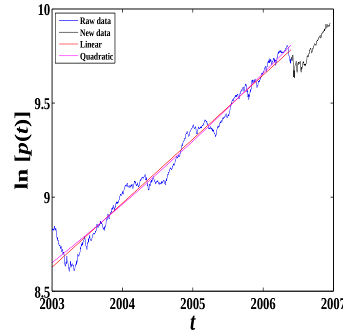


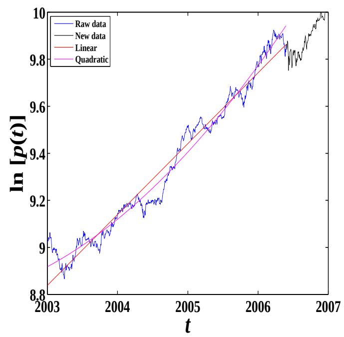
(a) Stock No. 1: J203



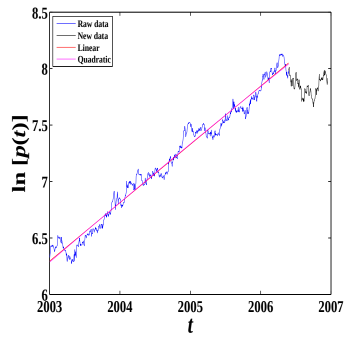
(b) Stock No. 2: J210



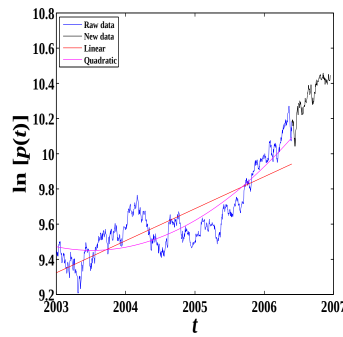
(c) Stock No. 3: J257



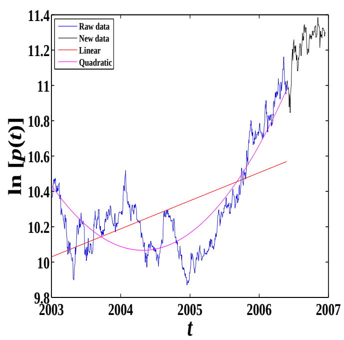
(d) Stock No. 4: J580



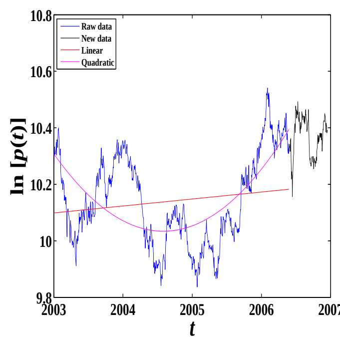
(e) Stock No. 5: ABL



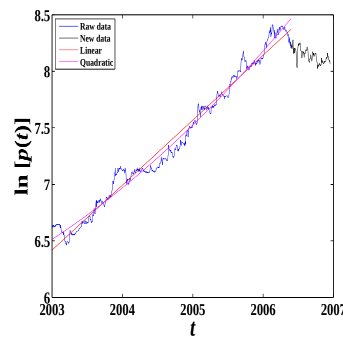
(f) Stock No. 6: AGL



(g) Stock No. 7: AMS



(h) Stock No. 8: ANG



(i) Stock No. 9: APN

Fig. 11. Linear fits and quadratic fits of the prices of stocks from No. 1 to No. 9.

Table 5. The 45 financial time series analyzed. Rows in red are indices.

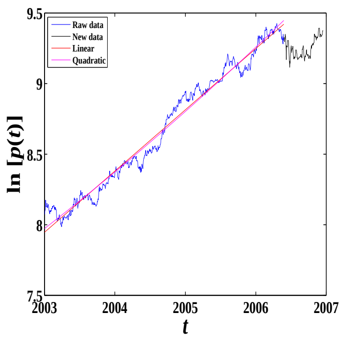
Ticker	Company name	Major Industry	Sub Industry
J203	JH-OVER	FTSE/JSE Africa All Share	
J210	JSE-RESI	FTSE/JSE Africa Resource 20	
J257	JH-ASIN	FTSE/JSE Africa All Share Industrials	
J580	JSE-FINA	FTSE/JSE Africa All Share Financial	
ABL	African Bank Investments Ltd	Financial	Commercial banks
AGL	Anglo American Plc	Metal producers & Products manufacturers	Miscellaneous metal producers
AMS	Anglo Platinum Ltd	Metal producers & Products manufacturers	Miscellaneous metal producers
ANG	Anglogold Ashanti Ltd	Metal producers & Products manufacturers	Gold producers
APN	Aspen Pharmacare Holdings Ltd	Drugs, cosmetics & health care	Diversified drugs, cosmetics, & health care
ASA	ABSA Group Ltd	Financial	Commercial banks
BAW	Barloworld Limited	Diversified	General diversified
BIL	BHP Billiton Plc	Metal producers & Products manufacturers	Diversified metal producers
BVT	Bidvest Group Ltd	Diversified	General diversified
ECO	Edgars Consolidated Stores Ltd	Retailers	Apparel store chains
FSR	FirstRand Ltd	Financial	Commercial banks

Table 6. The 45 financial time series analyzed (*Continued*).

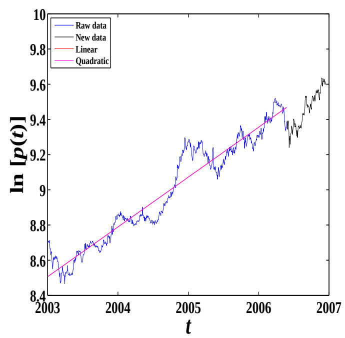
Ticker	Company name	Major Industry	Sub Industry
GFI	Gold Fields Ltd	Metal producers & Products manufacturers	Gold producers
HAR	Harmony Gold Mining Company Ltd	Metal producers & Products manufacturers	Gold producers
IMP	Impala Platinum Holdings Ltd	Metal producers & Products manufacturers	Miscellaneous metal producers
INL	Investec Ltd	Financial	Commercial banks
INP	Investec Ltd	Financial	Commercial banks
IPL	Imperial Holdings Ltd	Transportation	Other transportation
JDG	JD Group Ltd	Retailers	Miscellaneous retailers
KMB	Kumba Resources Ltd	Metal producers & Products manufacturers	Diversified metal producers
LBT	Liberty International Plc	Financial	Land and real estate
LGL	Liberty Group Ltd	Financial	Insurance companies
MLA	Mittal Steel South Africa Ltd	Metal producers & Products manufacturers	Diversified metal producers
MTN	MTN Group Ltd	Utilities	Telecommunications
NED	Nedbank Group Ltd	Financial	Commercial banks
NPN	Naspers Ltd	Recreation	Radio & TV broadcasts
NTC	Network Healthcare Holdings Ltd	Miscellaneous	Medical services

Table 7. The 45 financial time series analyzed (*Continued*)

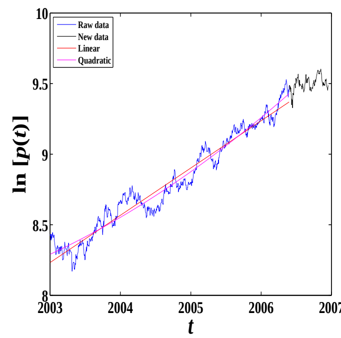
Ticker	Company name	Major Industry	Sub Industry
OML	Old Mutual plc	Financial	Insurance companies
PIK	Pick n Pay Stores Ltd	Retailers	Miscellaneous retailers
PPC	Pretoria Portland Cement Company Ltd	Construction	Cement producers
RCH	Richemont Securities Ag	Clothing & Footware	
REM	Remgro Ltd	Diversified	General diversified
RMH	RMB Holdings Ltd	Financial	Other financial services
SAB	SABMiller plc	Food & beverages	Brewers
SAP	Sappi Ltd	Paper	Diversified paper
SBK	Standard Bank Group Ltd	Financial	Commercial banks
SHF	Steinhoff International Holdings Ltd	Miscellaneous	Furnishings
SLM	Sanlam Ltd	Financial	Insurance companies
SOL	Sasol Ltd	Chemicals	Diversified chemical mfrs.
TBS	Tiger Brands Ltd	Diversified	General diversified
TKG	Telkom SA Ltd	Utilities	Telecommunications
WHL	Woolworths Holdings Ltd	Retailers	Miscellaneous retailers



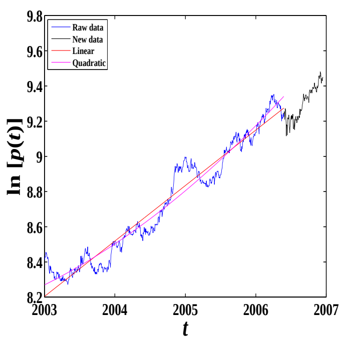
(a) Stock No.10: ASA



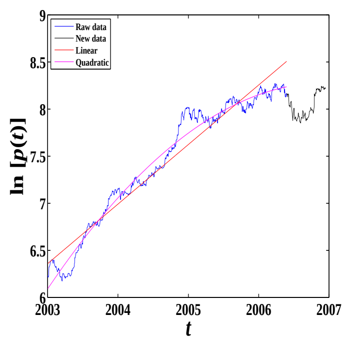
(b) Stock No.11: BAW



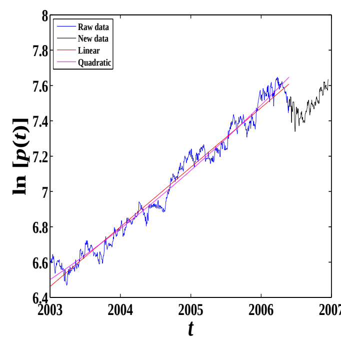
(c) Stock No.12: BIL



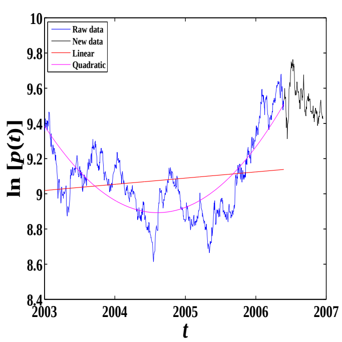
(d) Stock No.13: BVT



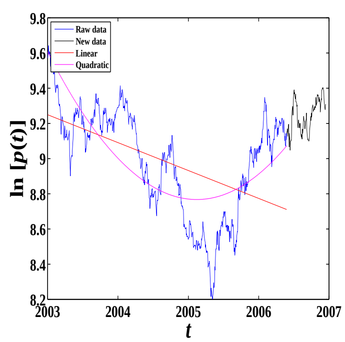
(e) Stock No.14: ECO



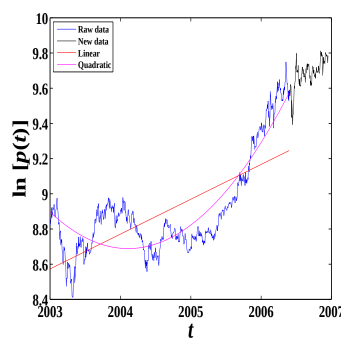
(f) Stock No.15: FSR



(g) Stock No.16: GFI

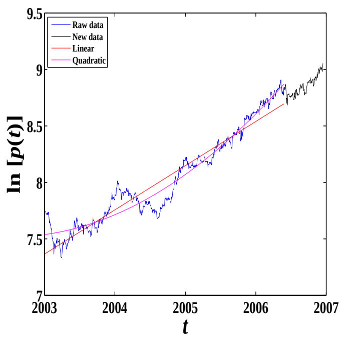


(h) Stock No.17: HAR

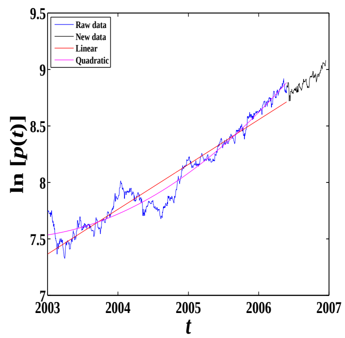


(i) Stock No.18: IMP

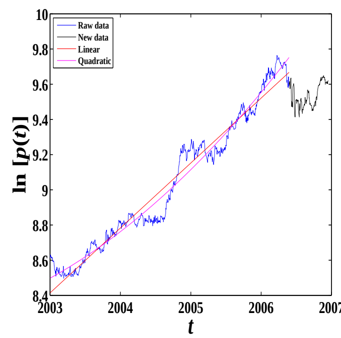
Fig. 12. Linear fits and quadratic fits of the prices of stocks from No.10 to No.18.



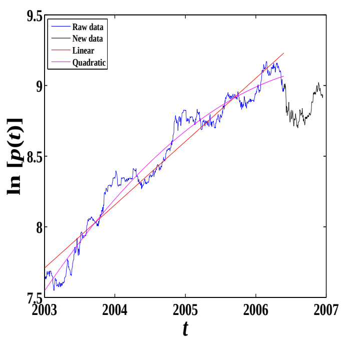
(a) Stock No.19: INL



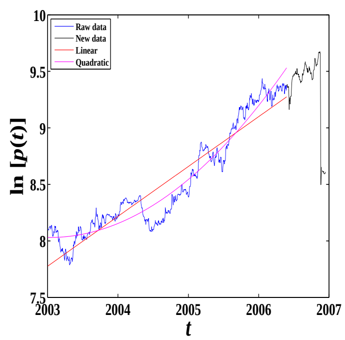
(b) Stock No.20: INP



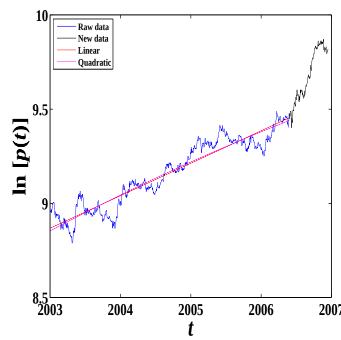
(c) Stock No.21: IPL



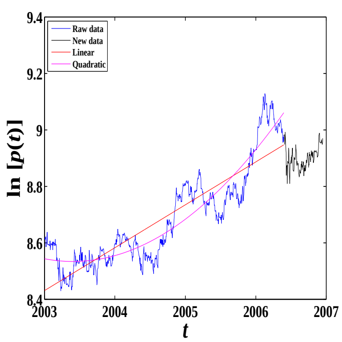
(d) Stock No.22: JDG



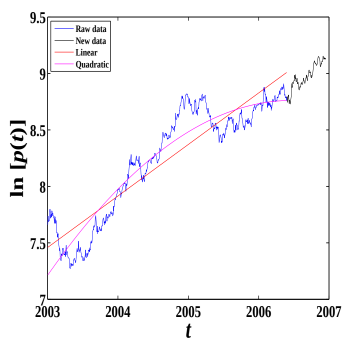
(e) Stock No.23: KMB



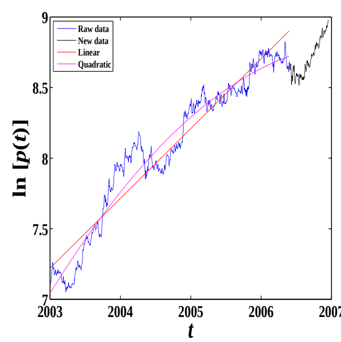
(f) Stock No.24: LBT



(g) Stock No.25: LGL

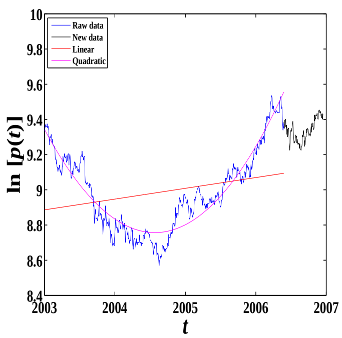


(h) Stock No.26: MLA

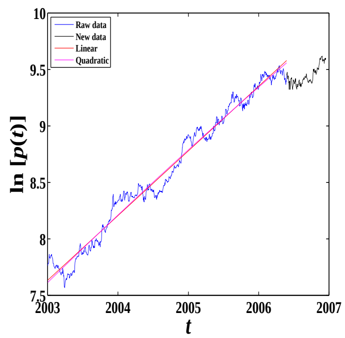


(i) Stock No.27: MTN

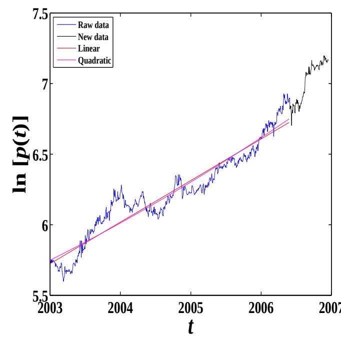
Fig. 13. Linear fits and quadratic fits of the prices of stocks from No.19 to No.27.



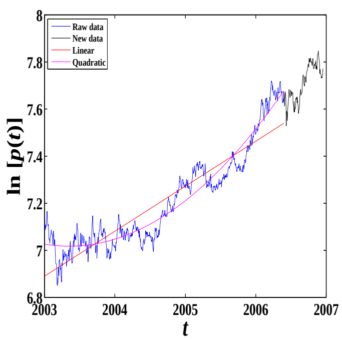
(a) Stock No.28: NED



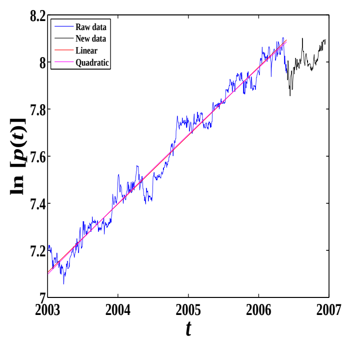
(b) Stock No.29: NPN



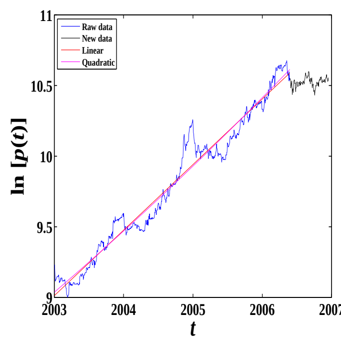
(c) Stock No.30: NTC



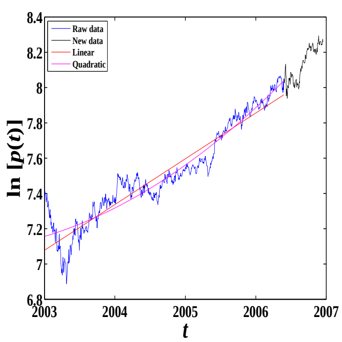
(d) Stock No.31: OML



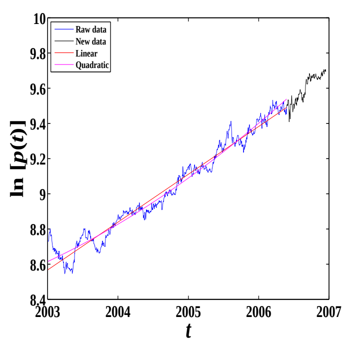
(e) Stock No.32: PIK



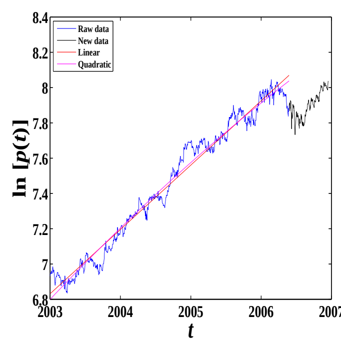
(f) Stock No.33: PPC



(g) Stock No.34: RCH

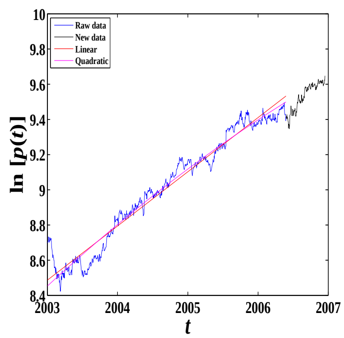


(h) Stock No.35: REM

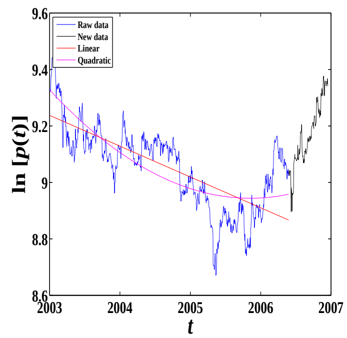


(i) Stock No.36: RMH

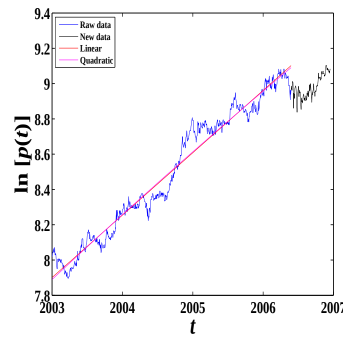
Fig. 14. Linear fits and quadratic fits of the prices of stocks from No.28 to No.36.



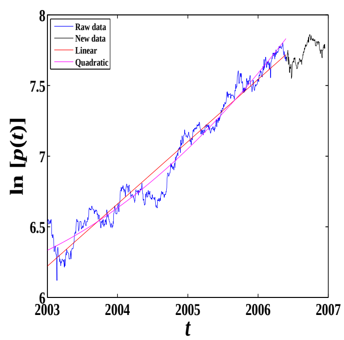
(a) Stock No.37: SAB



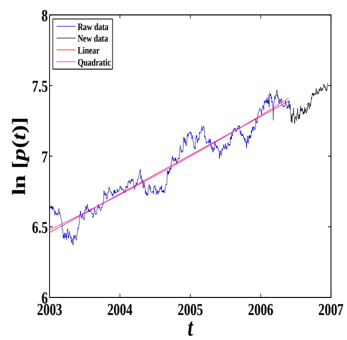
(b) Stock No.38: SAP



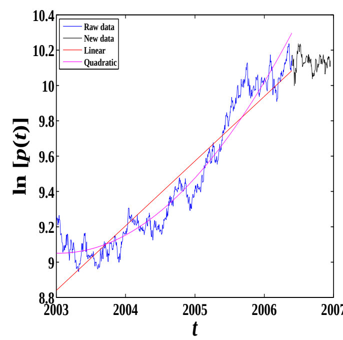
(c) Stock No.39: SBK



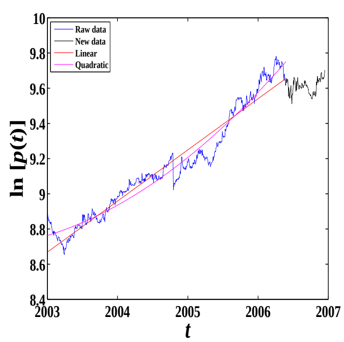
(d) Stock No.40: SHF



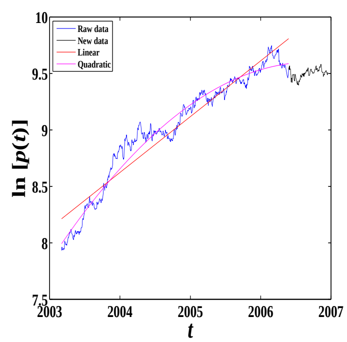
(e) Stock No.41: SLM



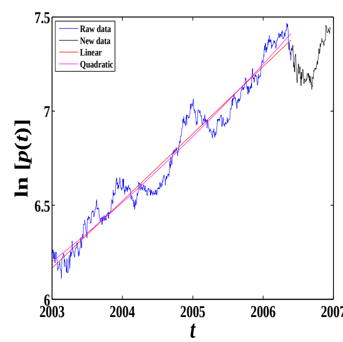
(f) Stock No.42: SOL



(g) Stock No.43: TBS

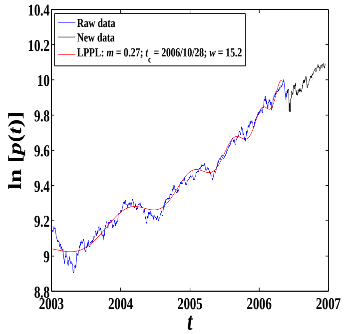


(h) Stock No.44: TKG

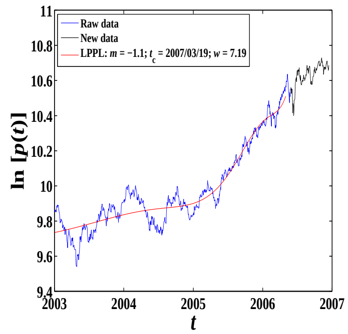


(i) Stock No.45: WHL

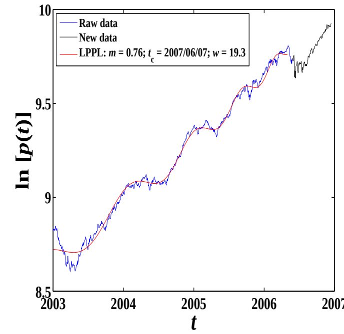
Fig. 15. Linear fits and quadratic fits of the prices of stocks from No.37 to No.45.



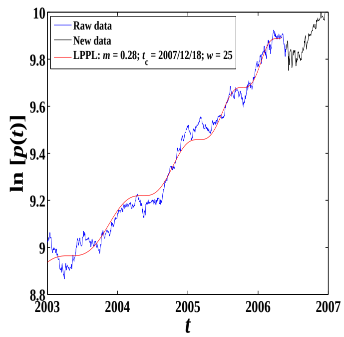
(a) Stock No. 1: J203



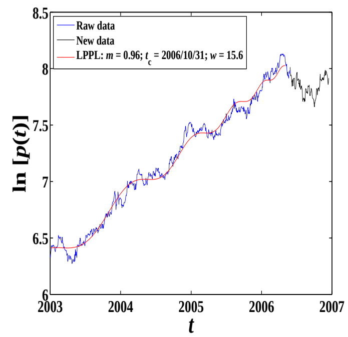
(b) Stock No. 2: J210



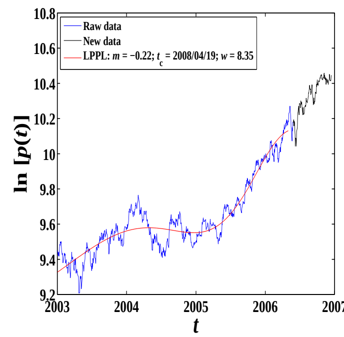
(c) Stock No. 3: J257



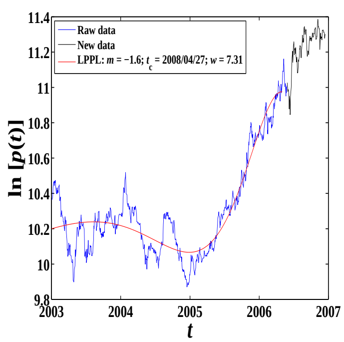
(d) Stock No. 4: J580



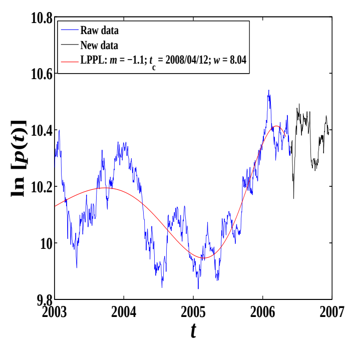
(e) Stock No. 5: ABL



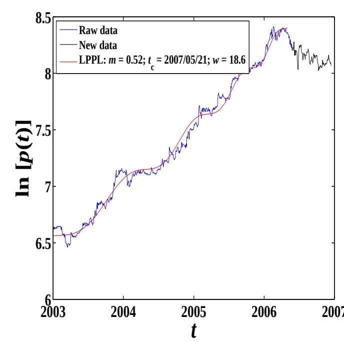
(f) Stock No. 6: AGL



(g) Stock No. 7: AMS

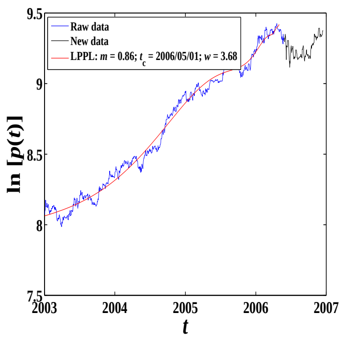


(h) Stock No. 8: ANG

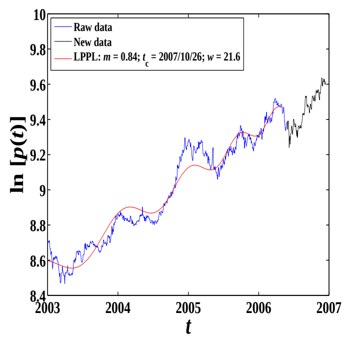


(i) Stock No. 9: APN

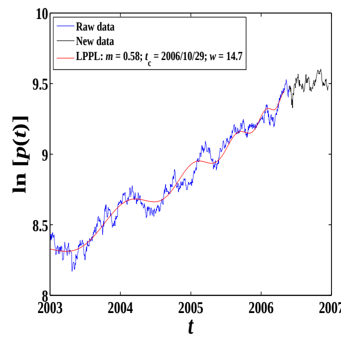
Fig. 16. Log-periodic power-law fits of the prices of stocks from No. 1 to No. 9.



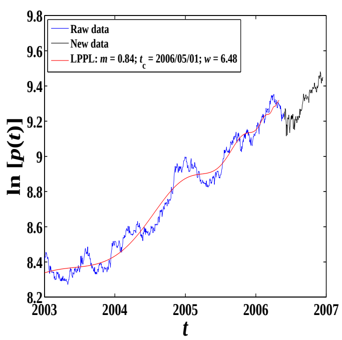
(a) Stock No.10: ASA



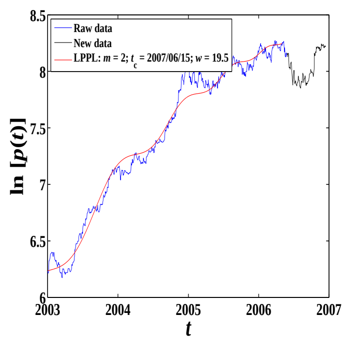
(b) Stock No.11: BAW



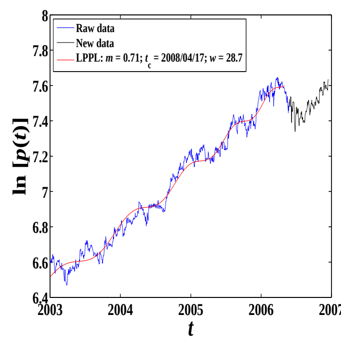
(c) Stock No.12: BIL



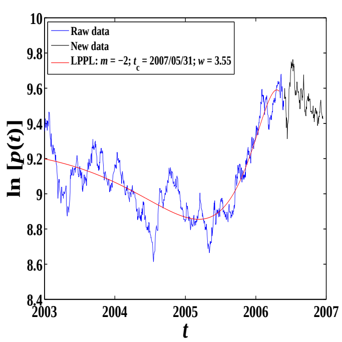
(d) Stock No.13: BVT



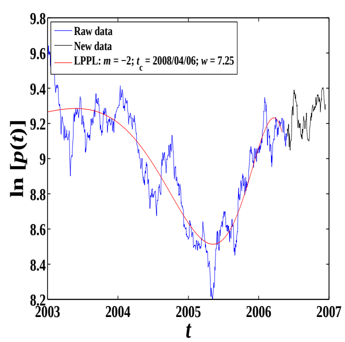
(e) Stock No.14: ECO



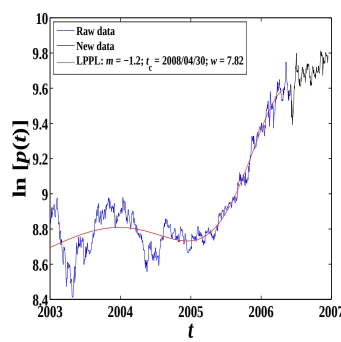
(f) Stock No.15: FSR



(g) Stock No.16: GFI

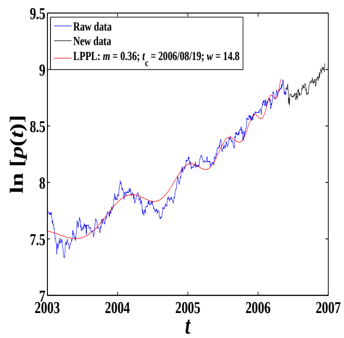


(h) Stock No.17: HAR

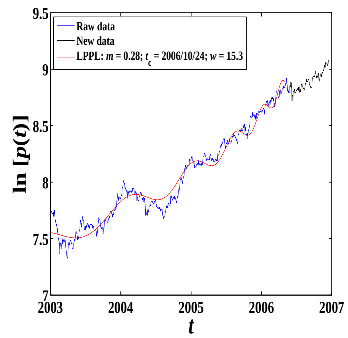


(i) Stock No.18: IMP

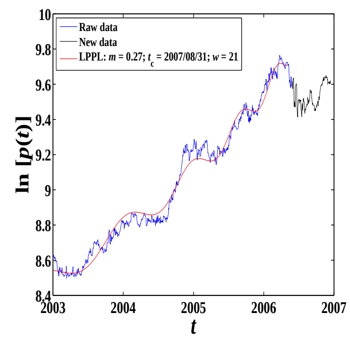
Fig. 17. Log-periodic power-law fits of the prices of stocks from No.10 to No.18.



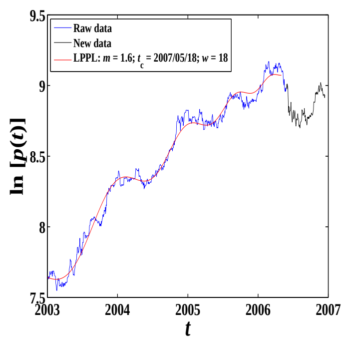
(a) Stock No.19: INL



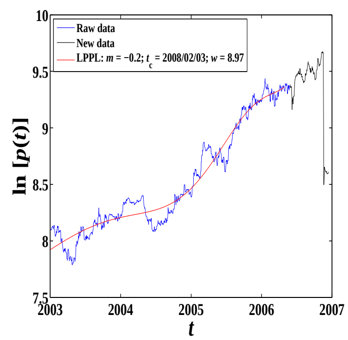
(b) Stock No.20: INP



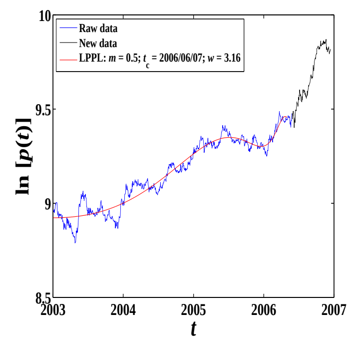
(c) Stock No.21: IPL



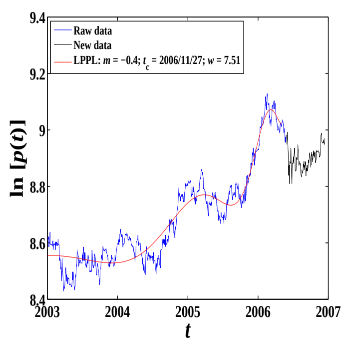
(d) Stock No.22: JDG



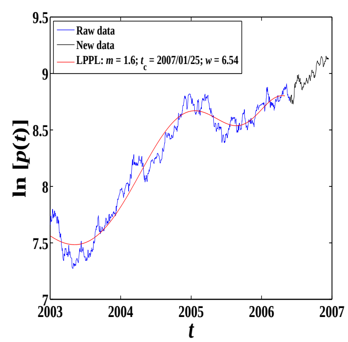
(e) Stock No.23: KMB



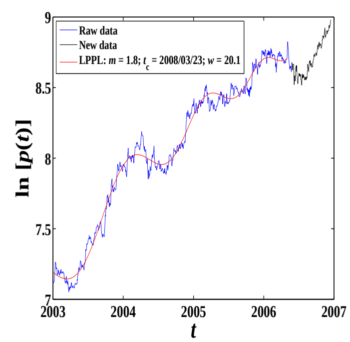
(f) Stock No.24: LBT



(g) Stock No.25: LGL

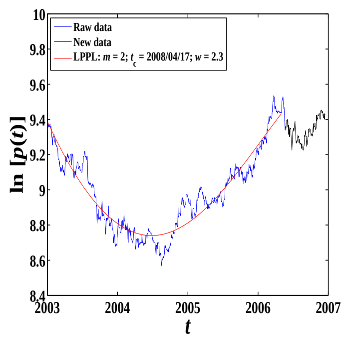


(h) Stock No.26: MLA

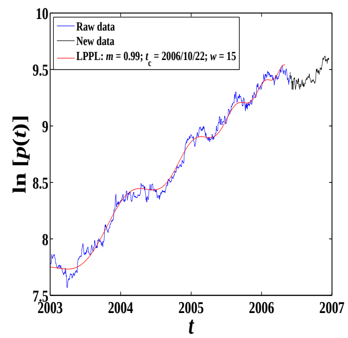


(i) Stock No.27: MTN

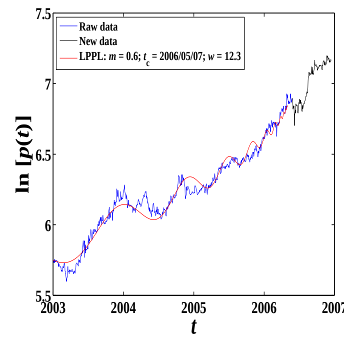
Fig. 18. Log-periodic power-law fits of the prices of stocks from No.19 to No.27.



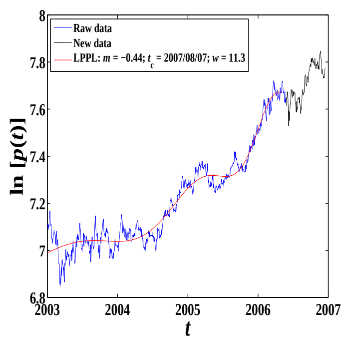
(a) Stock No.28: NED



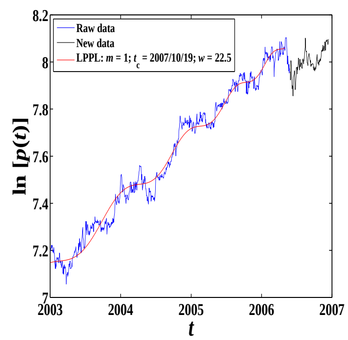
(b) Stock No.29: NPN



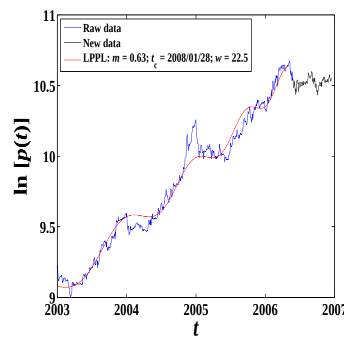
(c) Stock No.30: NTC



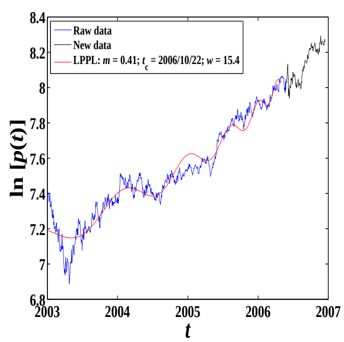
(d) Stock No.31: OML



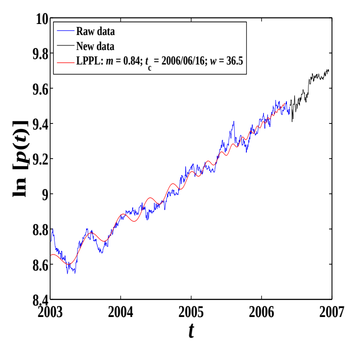
(e) Stock No.32: PIK



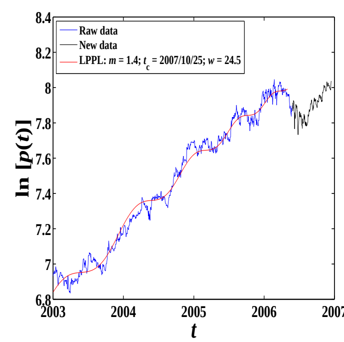
(f) Stock No.33: PPC



(g) Stock No.34: RCH

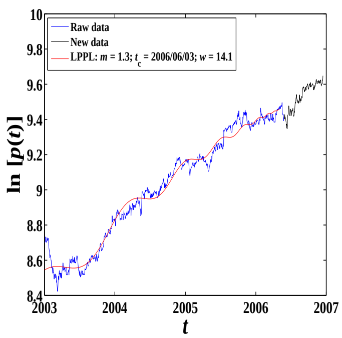


(h) Stock No.35: REM

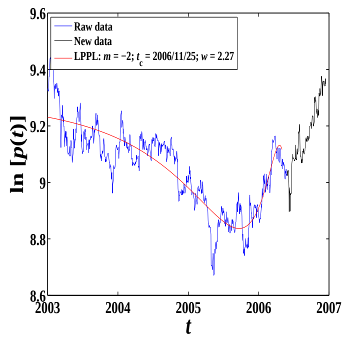


(i) Stock No.36: RMH

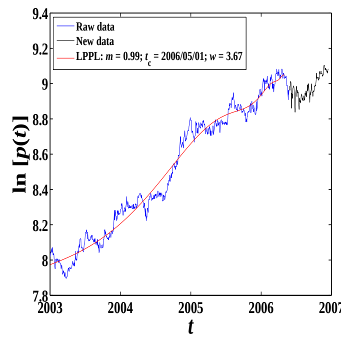
Fig. 19. Log-periodic power-law fits of the prices of stocks from No.28 to No.36.



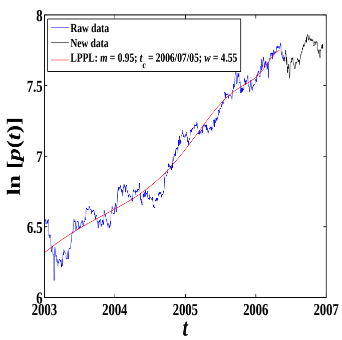
(a) Stock No.37: SAB



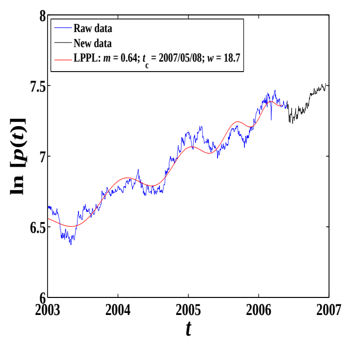
(b) Stock No.38: SAP



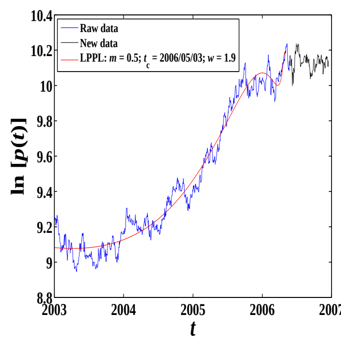
(c) Stock No.39: SBK



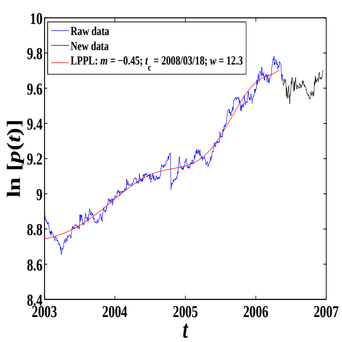
(d) Stock No.40: SHF



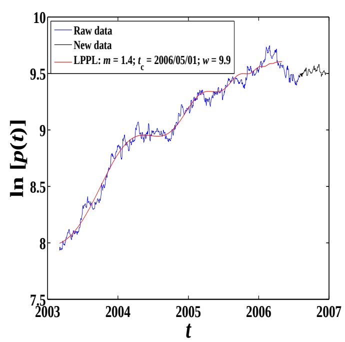
(e) Stock No.41: SLM



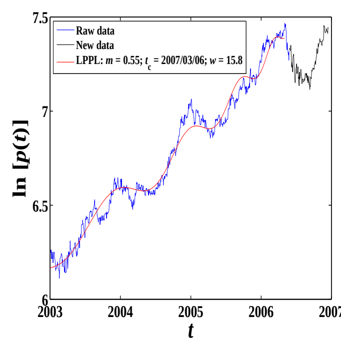
(f) Stock No.42: SOL



(g) Stock No.43: TBS



(h) Stock No.44: TKG



(i) Stock No.45: WHL

Fig. 20. Log-periodic power-law fits of the prices of stocks from No.37 to No.45.